A Fractal Comparison of Real and Austrian Business Cycle Models

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Abstract

This paper examines macromonetary data for fractal character and stochastic dependence. Lo's modified R/S test for stochastic dependence is used to explore the data's fractal properties along with five fractal analysis techniques for estimating the Hurst exponent, Mandelbrot-Lévy characteristic exponent, and fractal dimension. Techniques employed are rescaled-range analysis, power-spectral density analysis, roughness-length analysis, the variogram or structure function method, and wavelet analysis. Monthly data is analyzed over the 1959-2006 range and a quarterly dataset extending from 1947-2006 is also examined. Strong evidence is found for stochastic dependence in the monetary aggregates, which suggests an activist monetary policy, and for nominal consumption expenditures, supporting Austrian business cycle theory. No evidence for stochastic dependence in investment productivity is found, and this finding is invariant to whether productivity ratios are adjusted to account for credit expansion.

Introduction

This paper examines the distribution of changes in a vector of macromonetary data. Statistical tests focusing on five alternative methods for estimating Hurst (1951) exponent, fractal dimension, and Mandelbrot-Lévy characteristic exponent (Lévy 1925) are used. Lo's (1991) modified rescaled-range test for stochastic dependence is also used (Hays, Schreiber, et al 2000; Mulligan 2000a). The paper is organized as follows. A literature review is provided in the second section. The data are documented in the third section. Methodology and results are presented in the fourth and fifth sections. The conclusions are provided in the sixth section.

Mandelbrot's (1972a, 1975, 1977) and Mandelbrot and Wallis's (1969) R/S or rescaled range analysis characterizes time series as one of four types: 1.) dependent or autocorrelated series, 2.) persistent, trend-reinforcing series, also called biased random walks, random walks with drift, or fractional Brownian motion, 3.) random walks, or 4.) anti-persistent, ergodic, or mean-reverting series.

Mandelbrot-Lévy distributions are a general class of probability distributions derived from the generalized central limit theorem, and include the normal or Gaussian and Cauchy as limiting cases (Lévy 1925; Gnedenko and Kolmolgorov 1954). Samuelson (1982) popularized the term Mandelbrot-Lévy. The reciprocal of the Mandelbrot-Lévy characteristic exponent α is the Hurst exponent H, and estimates of H indicate the probability distribution underlying a time series. $H = 1/\alpha = 1/2$ for normally-distributed or Gaussian processes. $H = 1$ for Cauchy-distributed processes. $H = 2$ for the Lévy distribution governing tosses of a fair coin. H is also related to the fractal dimension D by the relationship $D = 2 - H$. Series with different fractal statistics exhibit different properties as described in Table 1.

Table 1
Fractal Taxonomy of Time Series

Term	'Color'	Hurst exponent (H)	Fractal dimension (D)	Characteristic exponent (α)
Antipersistent, Ergodic, Mean-reverting, Negative serial correlation, 1/f noise	Pink noise	$0 \le H < ½$	$0 \le D < 1.50$	$2.00 \le \alpha < \infty$
Gaussian process, Normal distribution	White noise	H º ½	D º 1.50	α º 2.00
Brownian motion, Wiener process	Brown noise	H º ½	D º 1.50	α º 2.00
Persistent, Trend-reinforcing, Hurst process	Black noise	$½ < H < 1$	$1 < D < 1.50$	$1.00 < \alpha < 2.00$
Cauchy process, Cauchy distribution	Cauchy noise	H º 1	D º 1	α º 1

Note: Brown noise or Brownian motion is the cumulative sum of a normally-distributed white-noise process. The changes in, or returns on, a Brownian motion, are white noise. Fractal statistics are the same for brown and white noise because the brown-noise process is differenced as part of the estimation process, yielding white noise.

Literature review

The search for long memory

in capital markets has been a fixture in the literature applying fractal geometry and chaos theory to economics since Mandelbrot (1963b) shifted his attention from income distribution to speculative prices. Fractal analysis has been applied extensively to equities (Green and Fielitz 1977; Lo 1991; Barkoulas and Baum 1996; Peters 1994, 1996; Koppl et al 1997; Kraemer and Runde 1997; Barkoulas and Travlos 1998; Koppl and Nardone 2001; Mulligan 2004, and Mulligan and Lombardo 2004), interest rates (Duan and Jacobs 1996; Barkoulas and Baum 1997a, 1997b; and Hays, Schreiber, et al 2000), commodities (Barkoulas, Baum, and Ogutz 1998), exchange rates (Cheung 1993; Byers and Peel 1996; Koppl and Yeager 1996; Barkoulas and Baum 1997c; Chou and Shih 1997; Andersen and Bollerslev 1997; Koppl and Broussard 1999, and Mulligan 2000a), and derivatives (Fang, Lai, and Lai 1994; Barcoulas, Labys, and Onochie 1997; and Corazza, Malliaris, and Nardelli 1997). Fractal analysis has also been applied to income distribution (Mandelbrot 1963a) and macroeconomic data (Peters 1994, 1996).

Gilanshah and Koppl (2005) advance the thesis that postwar money demand and monetary policy behavior were mostly stable from 1945-1970, but that instability emerged during the seventies as the Federal Reserve System adopted more activist policies and procedures.

If the Federal Reserve System becomes an active market player at times, the influence of this one "big player" would be to reduce the stability of money demand, as the many smaller players attempt to react to, as well as anticipate, big player moves. The smaller players' behavior should exhibit herding if it is difficult to anticipate or observe big player behavior, or if that behavior changes abruptly at the big player's discretion, and if it is relatively easy to observe behavior of other small players. If the Federal Reserve System is a big player acting in accordance with discretion as opposed to rules, the many little players would not appear to be following any coherent behavior, even if little players developed and followed consistent and rational strategic responses. Monetary aggregates would exhibit stochastic dependence over time, and this statistical behavior could spill over into other macroeconomic data processes.

Even if the little players respond according to set rules, because the big player acts unpredictably through discretion, the little players' behavior seems incoherent.

If this reading is correct, the instability in money demand is not a statistical artifact of specification error, and cannot be removed by adding variables to conventional money demand models.

As the Federal Reserve System began to adopt more activist policy measures during the 1970s, estimates generated by standard money demand specifications began to show sizable prediction errors. If activist monetary policy does indeed impose instability, this implies that the Federal Reserve System should abandon discretion and pursue money supply targets according to fixed rules. This implication runs counter to a prevailing inference presented in the literature on money demand instability. Mishkin's (1995:572) view is representative: "because the money demand function has become unstable, velocity is now harder to predict, and setting rigid money supply targets in order to control aggregate spending in the economy may not be an effective way to conduct monetary policy."

But as Gilanshah and Koppl (2005) argue, since the money demand instability results from Federal Reserve activism, the situation calls for less discretion, not more.

In their view, one mechanism introducing herding or bandwagon effects in money demand is cash managers' attempts to enhance their reputations, which enhances their job security and earning potential. Cash managers seek to enhance their reputations in a manner similar to, and for the same reasons as, portfolio managers (Scharfstein and Stein 1990). Cash managers achieve and maintain reputation through conformity with industry practice, a global criterion, and through conduct appropriate to the unique circumstances of their business enterprise, a local criterion.

Pursuit of the global criterion imposes herding behavior or bandwagon effects. If cash managers act as others do and things go well, their reputation is assured. If they act as others do and things go badly, the blame is shared throughout the profession. If cash managers defy prevailing practice in their profession and things go badly, their reputation is ruined. Sharfstein and Stein (1990:466) call this incentive to imitate standard practices the "sharing-the-blame effect." Similar behavior can be observed among the majority of less-innovative entrepreneurial planners.

If entrepreneurial planners defy the common wisdom and things go well, their reputation is strongly enhanced and they enjoy improved income prospects and job security. This is a powerful counter-incentive to herding. Not all entrepreneurial planners are constitutionally capable of acting independently of their peers, and some may require the security of the herd. Some entrepreneurs will herd; others will not. Big player conduct affects the fraction that herds. Activist monetary policy, particularly credit expansion, impairs the value of local information which could be exploited by the more independent entrepreneurs. Thus discretionary conduct by the monetary authorities promotes herding and introduces more volatility into macromonetary data.

Data

The data are monthly-observed monetary aggregates, macroeconomic expenditures, indexes, and ratios over the 1959-2006 range.

Macroeconomic data, specifically output measures and interest rates, are also examined over the same period to determine if their behavior appears significantly driven by the monetary data. Quarterly macromonetary processes were examined over the 1947-2006 period. Monetary series were extended from 1947-1958 from Gordon (1986:805-806).

GMB is the logarithmic first difference of the monetary base.

GM1 is the logarithmic first difference of M1.

GM2 is the logarithmic first difference of M2.

GM3 is the logarithmic first difference of M3.

GMZM is the logarithmic first difference of MZM.

GCUR is the logarithmic first difference of currency.

GRR is the logarithmic first difference of required reserves.

GER is the logarithmic first difference of excess reserves.

GCD is the logarithmic first difference of checkable deposits.

GTD is the logarithmic first difference of time deposits.

GR10Y is the first difference of the 10 year Treasury bond rate.

GR3M is the first difference of the 3 month Treasury bill secondary market rate.

GR is the first difference of the term spread, the 10 year rate minus the 3 month rate.

GIIP is the first difference of the index of industrial production.

GCN is the logarithmic first difference of nominal consumption expenditures.

GC is the logarithmic first difference of real consumption expenditures.

GCP is the first difference of the consumption expenditures deflator.

GDI is the logarithmic first difference of real disposable income.

GIN is the logarithmic first difference of nominal investment expenditures.

GI is the logarithmic first difference of real investment expenditures.

GP is the first difference of the GDP deflator.

RP is a productivity index computed as the ratio of nominal consumption expenditures divided by nominal investment expenditures. The productivity indices are difference-stationary so are not differenced for fractal analysis.

AP1 an ABC-adjusted productivity index computed as the ratio of nominal consumption expenditures minus the increase in M1 from the last period *divided*

by nominal investment expenditures plus the increase in M1 from the last period. As with RP, AP1 is difference-stationary.

AP2 is the first difference of a productivity index computed as above, but using M2 in place of M1.

APB is a productivity index computed as above, but using the monetary base in place of M1.

Time series which were already represented as interest rates, percentages, or ratios, were simply first differenced without taking logarithms. Productivity measures were not differenced because they were already difference-stationary.

Methodology

Long memory series exhibit non-periodic long cycles, or persistent dependence between observations far apart in time; i.e., observable patterns which tend to repeat.

Long memory or persistent series tend to reverse themselves less often than a purely random series. Thus, they display a trend, and are also called black noise, in contrast to purely random white noise. Persistent series have long memory in that events are correlated over long time periods. In contrast, short-term dependent time series include standard autoregressive moving average and Markov processes, and have the property that observations far apart exhibit little or no statistical dependence. R/S or rescaled range analysis distinguishes random from non-random or deterministic series. The rescaled range is the range divided (rescaled) by the standard deviation.

Seemingly random time series may be deterministic chaos, fractional Brownian motion (FBM), or a mixture of random and non-random components.

Conventional statistical techniques lack power to distinguish unpredictable random components from highly predictable deterministic components. R/S analysis evolved to address this difficulty. R/S analysis exploits the structure of dependence in time series irrespective of their marginal distributions, statistically identifying non-periodic cyclic long run dependence as distinguished from short dependence or Markov character and periodic variation (Mandelbot 1972a: 259-260). The difference between long-memory processes, also called non-periodic long cycles, and short-term dependence, is that each observation in long memory processes has a persistent effect, on average, on all subsequent observations, up to some horizon after which memory is lost, whereas in contrast, short-term dependent processes display little or no memory of the past, and what short-term dependence can be observed often diminishes with the square of the time elapsed. For equity prices, long memory can be observed when a stock follows a trend or repeats a cyclical movement, even though the cycles can have time-varying frequencies. Short-term dependence is indicated when there are no observable trends or patterns beyond a very short time span, and the impact of any outliers or extreme values diminishes rapidly over time.

Mandelbrot (1963a, 1963b) demonstrated all stationary series can be categorized in accordance with their Hurst exponent H. The Hurst exponent was introduced in the hydrological study of the Nile valley and is the reciprocal of the characteristic exponent alpha (Hurst 1951). Some series are persistent or black noise processes with ($0.50 < H < 1.00$). These less noisy series exhibit clearer trends and more persistence the closer H is to one. However, Hs very close to one indicate high risk of large, abrupt changes, e.g., $H = 1.00$ for the Cauchy distribution, the basis for the characteristic exponent test. This research used the approach of estimating the Hurst exponent for each series over the whole sample period by five alternative techniques, then testing for Gaussian character, and finally testing for stability of the Hurst exponent over two subsamples by R/S to examine whether the behavior of the data processes changed during the time studied.

Results

Many macromonetary series are anti-persistent or ergodic, mean-reverting, or pink noise processes with ($0.00 < H < 0.50$), indicating they are more volatile than a random walk. Pink noise processes are used to model dynamic turbulence. Ergodic or antipersistent processes reverse themselves more often than purely random series. Ergodicity, that is, H significantly below 0.50, indicate policy makers persistently over-react to new information, imposing more macroeconomic volatility than would maintain in the absence of policy, and never learn not to over-react.

This observed phenomenon is directly analogous to Mussa's (1984) disequilibrium overshooting, in which the market process of adjustment toward final equilbrium is unstable, and never quiets down. Hs significantly above 0.50 demonstrate macroeconomic data series are not random walks.

Table 2
Fractal Analyses of Macromonetary Monthly Data Processes 1959-2006
Estimated Hurst Exponent H, Various Methods
(Standard Errors in Parentheses)

variable	R/S	Power Spectrum	Roughness-length	Variogram	Wavelet
GMB	0.018	-0.392	-0.044	0.003	0.514
	(0.0194634)	(6.2957945)	(0.00269)	(1.1896007)	
GM1	-0.013	-0.404	-0.048	-0.006	0.851
	(0.0128657)	(11.1950188)	(0.0014695)	(2.8028963)	
GM2	0.075	-0.29	-0.030	0.006	0.767
	(0.0100815)	(13.3460836)	(0.0008523)	(1.3703301)	
GM3	0.049	-0.1605	-0.034	0.034	0.597
	(0.0171313)	(5.5036998)	(0.0009275)	(0.6806059)	
GMZM	0.176	-0.2285	0.032	0.040	0.545
	(0.0056194)	(5.624958)	(0.0005663)	(0.3498935)	
GCUR	-0.007	-0.4175	0.014	-0.010	0.819
	(0.0198405)	(12.1109514)	(0.0004026)	(2.8371727)	
GRR	0.098	-0.496	-0.056	0.001	0.748
	(0.0118314)	(6.1537476)	(0.0012239)	(0.5085489)	
GER	0.079	-1.0185	-0.093	-0.004	0.985
	(0.0106919)	(6.4823799)	(0.0048483)	(0.0637986)	
GCD	-0.012	-0.4115	-0.066	-0.008	0.872
	(0.0132301)	(10.5566654)	(0.0014590)	(2.2587965)	
GTD	0.279	-0.058	0.078	0.113	0.192
	(0.0040276)	(5.5141182)	(0.0010699)	(0.0619385)	
GR10Y	0.191	-0.4425	0.064	0.021	0.436
	(0.0154230)	(2.7973051)	(0.0002411)	(0.0531330)	
GR3M	0.241	-0.5015	0.119	0.024	0.361
	(0.0110211)	(5.6629744)	(0.0013522)	(0.0937309)	
GR	0.228	-0.663	0.083	0.016	0.553
	(0.0055240)	(8.0740194)	(0.0007747)	(0.0593133)	
GIIP	0.215	-0.304	0.089	0.034	0.255
	(0.0142341)	(4.0755129)	(0.0001330)	(0.0258054)	
GCN	0.110	-0.539	-0.005	-0.008	0.786
	(0.0040738)	(6.8441381)	(0.0004587)	(0.0303925)	
GC	0.132	-0.5175	0.006	-0.001	0.755
	(0.0044611)	(4.0951791)	(0.0002475)	(0.0305760)	
GCP	0.220	-0.165	-0.006	0.039	0.341
	(0.0056719)	(6.2382107)	(0.0004521)	(0.0955249)	

Note:
The Mandelbrot-Lévy characteristic exponent alpha is the reciprocal of the Hurst exponent H, thus alpha = 1/H. The fractal dimension D = 2 – H.

Table 3
Lo Test for Stochastic Dependence on Macromonetary Monthly Data Processes 1959-200

		Autoregressive Order								
	N	1	2	3	4	5	6	7	8	9
GMB	576	1.62955	1.74704	1.87159	1.97637	2.02415	1.9856	1.93953	1.91353	1.9191
		0.09502 *	0.05006 **	0.02359 **	0.01184 **	0.00850 **	0.01112 **	0.01518 **	0.01801 **	0.01737
GM1	575	1.84767	2.21345	2.2386	2.25967	2.34268	2.28693	2.30709	2.3193	2.2415
		0.02742 **	0.00207***	0.00169***	0.00143***	0.00072***	0.00114***	0.00097***	0.00087***	0.00165*
GM2	575	3.08061	3.21314	2.95373	2.79783	2.72089	2.60229	2.53809	2.4797	2.3805
		0.00000***	0.00000***	0.00000***	0.00001***	0.00002***	0.00007***	0.00013***	0.00022***	0.00052*
GM3	565	3.4289	3.27621	2.97688	2.7754	2.64299	2.51502	2.42511	2.34378	2.2524
		0.00000***	0.00000***	0.00000***	0.00001***	0.00005***	0.00016***	0.00035***	0.00071***	0.00151*
GMZM	575	1.82378	1.74877	1.62799	1.55318	1.51153	1.4679	1.44394	1.42206	1.3893
		0.03177 **	0.04957 **	0.09579 *	0.13889	0.1687	0.20481	0.22685	0.24838	0.2830
GCUR	575	1.99419	2.18628	2.3859	2.45972	2.42729	2.43622	2.39082	2.32109	2.3019
		0.01048 **	0.00256***	0.00049***	0.00026***	0.00034***	0.00032***	0.00047***	0.00086***	0.00101*
GRR	576	1.28968	1.43671	1.43007	1.43589	1.47097	1.4434	1.45324	1.46212	1.4354
		0.40616	0.23382	0.24035	0.23462	0.2021	0.22737	0.21811	0.20999	0.2350
GER	576	0.988	1.15463	1.24377	1.36766	1.56416	1.62141	1.71392	1.80396	1.9372
		0.83648	0.60324	0.47046	0.30766	0.13175	0.09908 *	0.06039 *	0.03583 **	0.01541
GCD	575	1.89645	2.28756	2.24288	2.24416	2.31028	2.24438	2.25419	2.26823	2.1760
		0.02013 **	0.00114***	0.00163***	0.00162***	0.00094***	0.00161***	0.00149***	0.00133***	0.00277*
GTD	565	4.09756	3.54408	3.18204	2.9273	2.72514	2.56345	2.4313	2.32089	2.2253
		0.00000***	0.00000***	0.00000***	0.00000***	0.00002***	0.00010***	0.00033***	0.00086***	0.00188*
GR10Y	576	1.54439	1.52775	1.53391	1.53711	1.52375	1.51733	1.52429	1.52307	1.5138
		0.14481	0.15656	0.15213	0.14987	0.15949	0.16428	0.1591	0.16	0.1669
GR3M	576	1.274	1.24959	1.25874	1.2697	1.27032	1.29647	1.34068	1.36586	1.3606
		0.42768	0.4621	0.44908	0.43367	0.4328	0.39702	0.34002	0.30976	0.3158
GR	576	0.71535	0.71662	0.71911	0.72688	0.74001	0.76082	0.78301	0.79361	0.7894
		0.99562	0.99549	0.99523	0.99434	0.99255	0.98885	0.98354	0.98042	0.9816
GIIP	575	1.80936	1.65349	1.53971	1.46127	1.40639	1.36101	1.32679	1.29502	1.2658
		0.03468 **	0.08385 *	0.14805	0.21075	0.26462	0.31546	0.35742	0.39896	0.4390
GCN	575	3.28662	3.34738	3.32592	3.28583	3.2224	3.11188	3.00656	2.90835	2.8219
		0.00000***	0.00000***	0.00000***	0.00000***	0.00000***	0.00000***	0.00000***	0.00000***	0.00001*
GCR	575	1.2466	1.30352	1.32011	1.3356	1.34475	1.32818	1.30218	1.27093	1.2458
		0.46639	0.38762	0.36596	0.34633	0.33503	0.35566	0.38939	0.43194	0.4675
GCP	575	4.63613	4.15359	3.81082	3.53646	3.30485	3.10431	2.93944	2.80386	2.6848
		0.00000***	0.00000***	0.00000***	0.00000***	0.00000***	0.00000***	0.00000***	0.00001***	0.00003*

Note:
The null hypothesis is no stochastic dependence. Probability levels given below the Lo test statistics. Statistical significance at the
, at the 1% level by *.

This section discusses and interprets the results of five alternative fractal analysis methods for measuring the Hurst exponent H presented in Table 2. All data are converted to first differences, losing one observation. Standard errors are given in parentheses. H is estimated first for monthly data over the 1959-2006 sample.

Five techniques for estimating the Hurst exponent are reported in this paper: 1.) Mandelbrot's (1972a) AR1 rescaled-range or R/S analysis; 2.) power spectral-density analysis; 3.) roughness-length relationship analysis; 4.) variogram analysis; and 5.) wavelet analysis:

1.) Rescaled-range or R/S analysis: R/S analysis is the traditional technique introduced by Mandelbrot (1972a). Hs estimated by this method are generally far from 0.50, suggesting non-Gaussian processes. The difference between estimated Hs and 0.50 is statistically significant over the whole sample range and both subsamples for each series examined. Hs are always below 0.50, indicating ergodicity or antipersistence, e.g., negative serial correlation meaning the data processes persistently overcorrect.

This measurable antipersistence or ergodicity demonstrates policy makers habitually overreact to new information, and never learn not to.

Hs different from 0.50 demonstrate the data series have not been random walks; nevertheless, this finding may be due to short-term dependence still present after taking AR1 residuals, or systematic bias due to information asymmetries, or both.

2.) Power spectral density analysis: Hs estimated by this technique also fall in the persistent range ($H < 0.50$). Spectral density often provides very large standard errors for H.

3.) Roughness-length relationship method: All Hs are significantly less than 0.50, indicating antipersistence.

4.) Variogram analysis: Variogram analysis supports antipersistence for all series.

5.) Wavelet analysis: This method was developed by Daubechies (1990), Beylkin (1992), and Coifman et al (1992). Wavelet H estimates indicate antipersistence or ergodicity ($H < 0.50$) for time deposits, the 10 year and 3 month interest rates, the index of industrial production, and the consumption price deflator. Results suggest trend-reinforcing, persistent character for all other variables, in contrast with the other techniques.

6.) Lo's (1991) modified R/S analysis: Hypothesis tests are reported in Table 3. Lo's technique does not provide an estimate for H and the null hypothesis of no stochastic dependence is necessary to lend any credence to long memory suggested by the other five methods.

Strong evidence of stochastic dependence extending at least one year is found for the monetary base, M1, M2, M3, MZM, currency, checkable deposits, time deposits, nominal consumption, and the consumption deflator. The Index of Industrial Production appears to be stochastically dependent, but only with a two-month lag.

Table 4
Fractal Analyses of Macromonetary Quarterly Data Processes 1947-2006
Estimated Hurst Exponent H, Various Methods
(Standard Errors in Parentheses)

variable	R/S	Power Spectrum	Roughness-length	Variogram	Wavelet
GDI	0.162	-0.5885	0.099	-0.032	0.351

	(0.0038666)	(4.31637)	(0.0001122)	(0.0258613)	
GCN	0.087	-0.499	0.127	-0.036	0.311
	(0.0013857)	(15.4324)	(0.0003282)	(0.0789028)	
GC	0.144	-0.4225	0.145	-0.003	0.229
	(0.0020723)	(3.91107)	(0.0000446)	(0.0504925)	
GIN	0.066	-0.6565	0.178	-0.023	0.402
	(0.0044745)	(5.94697)	(0.0000314)	(0.0809408)	
GI	0.078	-0.769	0.171	-0.018	0.336
	(0.0035645)	(7.35241)	(0.0000105)	(0.0726327)	
GP	0.418	0.248	0.395	0.245	0.243
	(0.0013298)	(4.95443)	(0.0018726)	(0.0106749)	
GM1	0.018	-0.0585	0.188	0.124	0.504
	(0.0139340)	(3.96447)	(0.0007898)	(0.0496912)	
GM2	0.137	-0.046	0.304	0.111	0.104
	(0.0093461)	(4.71824)	(0.0000533)	(0.0270818)	
GMB	0.177	-0.2795	0.206	0.018	0.083
	(0.0052624)	(6.16468)	(0.0000216)	(0.0176325)	
RP	0.301	0.2655	0.372	0.225	0.199
	(0.0281422)	(3.9015610)	(0.0011170)	(0.1853207)	
AP1	0.307	0.2575	0.362	0.219	0.234
	(0.0238342)	(4.8460107)	(0.0010277)	(0.1989056)	
AP2	0.305	0.2665	0.383	0.252	0.328
	(0.0199283)	(8.0910254)	(0.0001840)	(0.1033507)	
APB	0.295	0.249	0.376	0.211	0.191
	(0.0279218)	(4.3645687)	(0.0011405)	(0.2017907)	

Note:
The Mandelbrot-Lévy characteristic exponent alpha is the reciprocal of the Hurst exponent H, thus alpha = 1/H. The fractal dimension D = 2 – H.

Table 5
Lo Test for Stochastic Dependence on Macromonetary Quarterly Data Processes 1947-2006
(Probability Levels in Parentheses)

	Autoregressive Order							
	1	2	3	4	5	6	7	8
GDI	1.40673	1.38759	1.37816	1.41393	1.46503	1.49573	1.50748	1.52146
	0.26426	0.285	0.29558	0.25672	0.20737	0.18118	0.17184	0.16119
GNC	3.07276	2.72132	2.53354	2.42323	2.32137	2.2517	2.17212	2.10373
	0.00000***	0.00002***	0.00013***	0.00036***	0.00086***	0.00152***	0.00285***	0.00478
GC	1.17211	1.07542	1.03438	1.02329	1.01993	1.03555	1.04176	1.06216
	0.5768	0.72103	0.77803	0.79266	0.79702	0.77647	0.7681	0.73988
GNI	0.79473	0.75217	0.74163	0.76791	0.81151	0.87051	0.93236	0.99374
	0.98007	0.99053	0.9923	0.98732	0.97423	0.94429	0.89547	0.82966
GI	0.79473	0.75217	0.74163	0.76791	0.81151	0.87051	0.93236	0.99374
	0.98007	0.99053	0.9923	0.98732	0.97423	0.94429	0.89547	0.82966
GP	4.20451	3.48924	3.05428	2.75439	2.53518	2.36609	2.23081	2.11905
	0.00000***	0.00000***	0.00000***	0.00002***	0.00013***	0.00059***	0.00180***	0.00427
GM1	3.32571	2.89285	2.60312	2.4221	2.27724	2.15938	2.07015	1.99938
	0.00000***	0.00000***	0.00007***	0.00036***	0.00124***	0.00315***	0.00612 **	0.01011
GM2	3.46758	2.99392	2.69202	2.48458	2.32315	2.19482	2.09177	2.00902
	0.00000***	0.00000***	0.00003***	0.00021***	0.00084***	0.00239***	0.00522 **	0.00945
GMB	3.27019	2.8996	2.66144	2.48498	2.34316	2.23141	2.13708	2.05403
	0.00000***	0.00000***	0.00004***	0.00021***	0.00071***	0.00179***	0.00373***	0.00687
RBCPROD	2.7654	2.333	2.09299	1.94231	1.83737	1.75876	1.6953	1.64139
	0.00001***	0.00078***	0.00518 **	0.01490 **	0.02923 **	0.04679 **	0.06694 *	0.08935
ABCPROD(M1)	3.00105	2.52607	2.25997	2.09153	1.97312	1.88391	1.81197	1.75144
	0.00000***	0.00014***	0.00142***	0.00523 **	0.01211 **	0.02182 **	0.03414 **	0.04881
ABCPROD(M2)	3.44684	2.88252	2.55647	2.34426	2.19134	2.07343	1.97766	1.89731
	0.00000***	0.00000***	0.00011***	0.00071***	0.00246***	0.00597 **	0.01174 **	0.02002
ABCPROD(MB)	2.79195	2.35869	2.1195	1.97094	1.86826	1.79163	1.72937	1.67584
	0.00001***	0.00063***	0.00425***	0.01229 **	0.02410 **	0.03857 **	0.05537 *	0.07442

Note:
The null hypothesis is no stochastic dependence. Probability levels given below the Lo test statistics. Statistical significance at the 10% level indicated by *, at the 5% level **, at the 1% level by ***.

Five estimates of the Hurst exponent for quarterly variables over the 1947-2006 range are presented in Table 4. R/S analysis suggests antipersistence for all series.

Power spectral density functions indicate the GDP deflator, M1, and M2 are persistent, but that all other series are antipersistent, though standard errors are too high to allow much confidence in these estimates. Roughness-length and variogram estimates indicate antipersistence for all series.

Wavelets suggest all series are antipersistent except M1which is either nearly normal or slightly persistent. Lo tests are reported in Table 5 with up to eight quarters of autoregressive processes removed. Results indicate stochastic persistence for nominal consumption, M1, M2, the monetary base, the GDP deflator, and for all productivity ratios, for up to eight quarters.

Conclusion

The logarithmic differences of macroeconomic data for a stable and growing economy should have Hurst exponents approximately equal to 0.50, indicating these series change in a purely random, normally-distributed manner. Series with long-term trends and non-periodic cycles should display time persistence with $H > 0.50$, unless economic efficiency imposes randomness and normality anyway. Much of the macroeconomic data in this study yield strong evidence of antipersistence, ergodicity, or negative serial correlation.

The conclusion suggested is that decision makers are incapable of correctly evaluating economic data, persistently overreact to the arrival of new information, and never learn not to overreact, and this behavioral shortcoming applies to monetary policy makers, money managers, and entrepreneurial planners.

A scenario rendering this finding more intuitive is that information relevant to a nation's macroeconomic performance arrives frequently, in infinitesimal increments, and seemingly at random. Decision makers habitually ignore the vast majority of this information, because the vast majority is unimportant or irrelevant, until it accumulates a critical mass they must finally recognize.

Then, perceiving they have ignored a body of relevant information which they have allowed to accumulate, they attempt to compensate for their history of informational sloth by overreacting. The expression "informational sloth" can just as validly be characterized as "filtering out noise."

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Glossary of Fractal Analysis Terms
ARFIMA process – autoregressive fractally integrated moving average process, represented by ARFIMA(p,d,q), where, as with an ARIMA(p,d,q) process, p represents the order of autoregression process, d the order of integration (i.e., the number of times the process must be differenced to impose stationarity), and q the order of the moving average process. An ARFIMA process differs from ARIMA in that the order of integration is not constrained to be an integer.
Antipersistence – a series that reverses itself more often than a purely random series, also called pink noise, ergodicity, 1/f noise, or negative serial correlation. (Peters 1994: 306).
Black noise – a series that reverses itself less often than a purely random series, displaying trends or repetitive patterns over time, also called persistence, positive serial correlation, or autocorrelation. (Peters 1994: 183-187).
Brown noise – the cumulative sum of a normally-distributed random variable, also called Brownian motion. (Peters 1994: 183-185; Osborne 1959).
Efficient Market Hypothesis – the proposition that market prices fully and correctly reflect all relevant information. A market is described as efficient with respect to an information set if prices do not change when the information is revealed to all market participants. There are three levels of market efficiency: weak, semi-strong, and strong. (Fama et al 1969; Malkiel 1987).
Lo test – the statistical test of the null hypothesis of no stochastic dependence based on computation of Q_n, a modified rescaled range statistic (Lo 1991).
Long memory – the property that any value a series takes on generally has a long and persistent effect, e.g., extreme values that repeat at fairly regular intervals. (Peters 1994: 274).
Multifractal Model of Asset Returns (MMAR) – a very general model of asset pricing behavior allowing for long-memory and fat-tailed distributions. Instead of infinite-variance distributions such as the Mandelbrot-Lévy and Cauchy distributions, the MMAR relies on fractional combinations of random variables with non-constant mean and variance, providing many of the properties of infinite-variance distributions. (Mandelbrot, Fisher, and Calvet 1997).
Non-periodic long cycles - a characteristic of long-memory processes, i.e., of statistical processes where each value has a long and persistent impact on values that follow it, that identifiable patterns tend to repeat over similar, though irregular, cycles (non-periodic cycles.) Also called the Joseph effect. (Peters 1994: 266).
Non-stationarity – the property that a series has a systematically varying mean and variance. Any series with a trend, e.g., U.S. GDP, has a growing mean and therefore is non-stationary. Brown-noise processes are non-stationary, but white-noise processes are stationary. (Granger 1989: 58).
Persistence or persistent dependence – a series that reverses itself less often than a purely random series, and thus tends to display a trend, also called black noise. Persistent series have long memory in that events are correlated over long time periods, and thus display non-periodic long cycles. (Peters 1994: 310).
Semi-strong-form Market Efficiency – the intermediate form of the efficient market hypothesis, asserting that market prices incorporate all publicly available information, including both historical data on the prices in question, and any other relevant, publicly-available data, and thus it is impossible for any market participant to gain advantage and earn excess profits, in the absence of inside information. (Peters 1994: 308).
Short-term dependence - the property that any value a series takes on generally has a transient effect, e.g., extreme values bias the series for a certain number of observations that follow. Eventually, however, all memory of the extreme event is lost, in contrast to long-memory or the Joseph effect. Special cases include Markov processes and serial correlation. (Peters 1994: 274).
Spectral density or Power-spectral Density Analysis – a fractal analysis based on the power spectra calculated through the Fourier transform of a series. (Peters 1994: 170-171).
Stationarity – the property that a series has a constant mean and variance. White, pink, and black-noise processes are all stationary. Because it is the cumulative sum of a white-noise process, a brown-noise process is non-stationary. (Granger 1989: 58).
Stochastic dependence – the property that time series exhibit non-periodic cycles. Behavior will be persistent for an unpredictable period (the Joseph effect) but may also be washed out, reversed, changed, or memory may be lost by some randomly-occuring event (the Noah effect.)
Strong-form Market Efficiency – the most restrictive version of the efficient market hypothesis, asserting that all information known to any one market participant is fully reflected in the price, and thus insider information provides no speculative advantage and cannot offer above average returns. (Malkiel 1987: 120).
Weak-form Market Efficiency – the least restrictive version of the efficient market hypothesis, asserting that current prices fully reflect the historical sequence of past prices. One implication is that investors cannot obtain above-average returns through analyzing patterns in historical data, i.e., through technical analysis. Also referred to as the Random Walk Hypothesis. One common way of testing for weak-form efficiency is to test price series for normality, however, normality is a sufficient rather than a necessary condition. (Malkiel 1987: 120).

White noise – a perfectly random process exhibiting no serial dependence. Normal processes meet this requirement, and normality is often conflated with white noise. Normality is a sufficient condition rather than a necessary condition for white noise. (Peters 1994: 312).

Appendix
Statistical methodology

Lo's modified Rescaled-range: Lo (1991) developed the adjusted R/S statistic, Q_n, replacing the denominator of the R/S with the square root of the sum of the sample variance and weighted covariance terms, and which has the property that its statistical behavior is invariant over a general class of short memory processes but deviates for long memory processes. Lo (1991) and Lo and MacKinlay (1988) used this technique to find little evidence of long memory in stock prices. Lo's Q_n has the advantage that it is robust against short-term dependence, provided Lo's underlying assumptions are satisfied.

Statistically significant Q_n's indicate rejection of the null hypothesis of no long-term dependence or long memory. The largest lag order of statistically significant Q-statistics gives the average non-periodic cycle length, beyond which most long memory (memory of initial conditions) is lost.

Rescaled-range or R/S analysis: R/S analysis is the conventional method introduced by Mandelbrot (1972a). Time series are classified according to the estimated value of the Hurst exponent H, which is defined from the relationship

$$R/S = an^H$$

where R is the average range of all subsamples of size n, S is the average standard deviation for all samples of size n, a is a scaling variable, and n is the size of the subsamples, which is allowed to range from an arbitrarily small value to the largest subsample the data will allow. Putting this expression in logarithms yields

$$\log(R/S) = \log(a) + H \log(n)$$

which is used to estimate H as a regression slope. Standard errors are given in parentheses. H ranges from 1.00 to 0.50 for persistent series, is exactly equal to 0.50 for random walks, ranges from zero to 0.50 for anti-persistent series, and is greater than one for a persistent or autocorrelated series.
Mandelbrot, Fisher, and Calvet (1997) refer to H as the self-affinity index or scaling exponent.

Power spectral density analysis:
This method uses the properties of power spectra of self-affine traces, calculating the power spectrum P(k) where k = 2p/*l* is the wavenumber, and *l*
is the wavelength, and plotting the logarithm of P(k) versus log(k), after applying a symmetric taper function which transforms the data smoothly to zero at both ends.
If the series is self-affine, this plot follows a straight line with a negative slope –b, which is estimated by regression and reported as beta, along with its standard error. This coefficient is related to the fractal dimension by: D = (5 - beta)/2. H and alpha are computed as H = 2 – D, and alpha = 1/H.
Power spectral density is the most common technique used to obtain the fractal dimension in the literature, although it is also highly problematic due to spectral leakage.

Roughness-length relationship method:

This method is similar to R/S, substituting the root-mean-square (RMS) roughness s(w) and window size w for the standard deviation and range. Then H is computed by regression from a logarithmic form of the relationship $s(w) = w^H$. As noted above, the roughness-length method provides standard errors so low the null hypothesis of H = 0.500 is nearly always rejected no matter how nearly normal the asset returns.

Variogram analysis:

The variogram, also known as variance of the increments, or structure function, is defined as the expected value of the squared difference between two y values in a series separated by a distance w. In other words, the sample variogram V(w) of a series y(x) is measured as: $V(w) = [y(x) – y(x+w)]^2$, thus V(w) is the average value of the squared difference between pairs of points at distance w . The distance of separation w is also referred to as the lag. The Hurst exponent is estimated by regression from the relationship $V(w) = w^{2H}$.

Wavelet analysis:

Wavelet analysis exploits localized variations in power by decomposing a series into time frequency space to determine both the dominant modes of variability and how those modes vary in time. This method is appropriate for analysis of non-stationary traces such as asset prices, i.e. where the variance does not remain constant with increasing length of the data set. Fractal properties are present where the wavelet power spectrum is a power law function of frequency. The wavelet method is based on the property that wavelet transforms of the self-affine traces also have self-affine properties.

Consider n wavelet transforms each with a different scaling coefficient a_i, where $S_1, S_2,..., S_n$ are the standard deviations from zero of the scaling coefficients a_i. Then define the ratio of the standard deviations $G_1, G_2, ..., G_{n-1}$ as: $G_1 = S_1/S_2$, $G_2 = S_2/S_3, ..., G_{n-1} = S_{n-1}/S_n$. Then the average value of G_i is estimated as $G_{avg} = (G_i)/(n – 1)$. The estimated Hurst exponent H is computed as a heuristic function of G_{avg}.

The Benoit software computes H based on first three dominant wavelet functions, i.e., n is allowed to vary up to 4, and i for the scaling coefficient a_i is allowed to vary from i = 0, 1, 2, 3.